EXTENDING STUDENTS IN MATHEMATICS

HOW TO EFFECTIVELY DIFFERENTIATE A MATHEMATICS PROGRAM TO CATER TO HIGH ACHIEVING STUDENTS

SECTION 1- CONTEXT

ESTABLISHING CONTENT AND CONTEXT FOR STUDENT LEARNING.

1A: THE STUDENTS

GRADE: 6
SUBJECT: Mathematics/Algebra
NUMBER OF STUDENTS: 7

The students were chosen because they are working above the expected level in Mathematics. This is evidenced by their reports and sample assessments shown in Appendix 4 (4a and 4e). The students need to be supported in extending their knowledge by learning new mathematics concepts and applying them to real life situations. For the purpose of this inquiry I will be specifically reporting on Students 1 and 2.

I selected Student 1 because he is motivated and enjoys mathematics. He responds well to being challenged in individual and small group settings. Due to this I think he will benefit from the inquiry as it will cater to his style of learning and through scaffolding will extend and challenge him.

Student 2 was selected because he achieves highly in mathematics but struggles to motivate himself. This often results in work that does not accurately reflect his understanding. Student 2 speaks English as an Additional Language and migrated from Korea 2 years ago. He is confident with speaking English, but shows some trepidation when reading and writing in English. This can affect his performance on mathematics tasks that have worded problems.

1B: CONTEXT OF STUDENT LEARNING

RANGE OF LEARNING LEVELS:

Within the focus group all received a 6.5 for mathematics in their latest report, which places them one year above the standard.

The range in the learning levels is not created by content knowledge, but the varying levels of motivation and focus among the students. The greatest difference in the learning levels can be seen in their ability to explain their learning. Three of the students, including Student 1, are able and comfortable with explaining their thinking and clarifying and refining their thought process. Four of the students are still learning and adapting to explaining their thinking process. They require support and encouragement in doing this.

FACTORS AFFECTING LEARNING:

English as an Additional Language

Students 2 and 4 are EAL students. Both are confident with their spoken language, but are still building confidence in their reading and writing. This has a considerable effect on their learning as they may avoid tasks that they perceive as text heavy. For example, mathematics projects or worded problems may cause them some stress because they are not confident with the language. I also need to be aware of this when assessing student understanding. One way I will cater for this is encouraging the students to use diagrams and numbers to show their working out. I will be designing tasks and using conferencing to ensure that students can show their mathematical
knowledge unimpeded by language barriers.

**Behavioural Aspects**

Within the group, several of the students find it difficult to motivate themselves to complete tasks to the best of their abilities. They have become accustomed to doing the bare minimum required to complete the task. I feel this is because I have not challenged them enough in their learning of Mathematics. To cater for this, I plan to engage the students through challenging learning tasks that include student voice.

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**1 C: CONTENT FOR THE PROGRAM OF LEARNING**

The action plan was developed with guidance from AusVELS, the Scope and Sequence of the school, the Term Planner and Weekly Planners. This was done to ensure the unit was relevant to the students and integrated with the rest of their education. It also ensured that the program accurately reflected the whole school approach to learning. Annotated versions of the guiding documents can be found in Appendix 1.

AusVELS provides progression points and proficiency standards that students are expected to reach each year. Both of these were consulted in the creation of the unit. Appendix 1a shows what progression points and proficiencies the students demonstrated and the learning goals that drove the program. The document also shows the understandings that the students had achieved by the end of the program.

The mathematics scope and sequence and term planners helped guide the unit, however I did not rely heavily on these documents as the content/concepts of the unit went beyond what was planned in these documents. From this point I used AusVELS to guide the inquiry.

The weekly planners were the working documents used to take notes, review and plan the next sessions. The annotations show session to session changes and learning outcomes. This was done on the weekly planner as well as the action plan because the weekly planner gives a holistic view of the week and is the main working document. These planners can be found in Appendix 1b.

When designing the unit it was important that it incorporate effective use of ICT. The year six students have a one to one iPad program. This gives them access to a powerful and effective tool for researching, formulating and communicating ideas. This was an intrinsic part of my planning.

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**1 D: PRIOR LEARNING OF STUDENTS**

To assess the prior learning of students I looked at three main sources. Firstly I analysed their work books. I looked for working out and application of mathematical concepts. What I found was that the students were able to get the right answer, but often showed no working out. Therefore it was not clear if the students could explain what they were doing. The workbooks also demonstrated that the students struggled with applying the concepts they had learnt to real life situations.

Due to the lack of explanations given in the workbooks I observed the students in class. This included talking to the students about what they were doing, encouraging them to explain their thinking process. The observation process confirmed the findings from their previous work. The students were not able to fully explain their thinking and struggled most with applying concepts to real life situations. However, this was not true of basic algebra knowledge. They were able to explain the significance of brackets and how they are used to manipulate sums. They could also explain why letters are used for variables and what a variable was. Observation notes can be found in Appendix 5.

I issued a pre-test to assess their knowledge of algebra. I used Mathletics tasks that were directly related to each point in AusVELS. The assessment showed that the students were between year 6.5 and 7.0 in terms of content knowledge. The results informed the learning outcomes for the program and the content to be covered. Copies of the pre-test can be found in Appendix 4a and 4e.
Overall it was clear that the students had strong content knowledge. They could explain and apply basic algebra processes, but struggled to apply more complex mathematical concepts.

**IE: LEARNING OUTCOMES FOR THE PROGRAM OF LEARNING**

The original outcomes were based on the students' prior knowledge in accordance with AusVELS to develop the students' knowledge from the understanding at 6.5 to achieving at 7 or 7.5. Specifically this meant that the students would be able to:

1. Extend and apply the distributive law to the expansion of algebraic expressions.
2. Factorise algebraic expressions by identifying numerical factors.
3. Simplify algebraic expressions involving the four operations.

These goals have a strong mathematical connection between them and flow into each other. Knowing the students capacity for new information I planned goals that allowed more growth on their completion.

As the unit developed, the learning outcomes were adapted from focusing on breadth to depth. The learning goals were changed as assessment showed that the students had a handle on these concepts but were not able to use them to problem solve. See section 3 for more detail on these changes. See Appendix 3 for notes on this.

**WHAT WILL BE DONE TO CATER FOR STUDENTS**

**STUDENTS WHO NEED DEVELOPING BEYOND THE LEARNING OUTCOMES:**

Part of developing student ability is giving them the opportunity to evaluate and create knowledge. Students who need developing beyond the learning outcomes will be encouraged to move from understanding and applying the new knowledge, to evaluating and creating ideas using algebra.

**STUDENTS WHO NEED SUPPORT TO MEET THE LEARNING OUTCOMES:**

In order to support students, I will cater to their learning styles and needs. I will do this by having individual conferences with the students to assess what I need to target and how best to convey it. I will use the information gathered during student conferences to guide my teaching and use of visual and physical tools. I will also give time for students to ask questions regarding the application and understanding of the concepts. This means some students will spend longer in the "we do it" phase of the Gradual Release Model.
SECTION 2: INQUIRY QUESTION

QUESTION FOR INQUIRY AND PROFESSIONAL LEARNING.

2A: THE QUESTION FOR INQUIRY

How to differentiate effectively, ensuring that high performing students are extended and receive adequate support in achieving their optimal learning outcomes?

I chose this question because I feel that I do not support high achieving students in extending themselves. I need to improve how I differentiate so that all students are catered for and targeted at the appropriate level. I find this particularly challenging in mathematics as I am not confident in this area.

2B: PROFESSIONAL LEARNING

OBSERVING THE PRACTICE OF OTHERS:

Date: 28.8.2014
Teachers Involved: J
What was seen/heard:

Overview:

I went to watch J teach a year 5 Mathematics lesson. I was interested in seeing how she differentiated her instruction to cater to all students in her classroom, particularly those that require extension. Jen’s lesson consisted of a mini-lesson followed by independent learning time, in which she conferenced with focus groups, teaching them new strategies.

Mini-Lesson:

The mini lesson consisted of introducing students to the task and then brainstorming strategies that could be used to solve the problem. During the mini-lesson J checked the students’ understanding by asking them where they were at with their understanding. The verbal check was effective and did not take away from the focus of the lesson.

Independent:

During the independent learning stage, J held focus groups responding to student needs. For example, where students were not sure how to convert between fractions and decimals she taught them how to do this in a small group on the floor. The result was that students were able to enter and exit the task at a level appropriate to their learning. This also meant that those that needed extension were given the opportunity to receive the targeted teaching.

Share time:

The share time was focused on strategies rather than content. This prompted the students to reflect on their learning rather than on the content of what they did. It also allowed the students to share their ideas and learn from each other. The share time was done from the students’ desks. This was useful as it made it a quick and targeted task.
What I learnt:

There are 3 main things I took away from this session:

1. Informally (verbally) checking students understanding is really useful for differentiation. It gives a snapshot of where students are at and allowed for small, targeted interventions within the lesson.
2. An effective way to differentiate is to have a task with many points of entry and that can support focus groups designed to challenge students and further their understanding.
3. Share time can be done as a quick activity from the students’ desks. This is useful as the students are not distracted by moving to the floor and can respond immediately to what they were doing.

How it helped address the question for inquiry:

Through the observation I learnt a new technique for holding focus groups. I had been planning focus groups in a formal way, rather than allowing student voice and choice to inform what focus groups to hold. I will use this model to not only extend students but cater to those that need more support.

Date: 3.9.2014
Teachers Involved: D

What was seen/heard:

Overview:

I observed D teaching my class algebra. He had a selection of problems for the students to solve. He structured the class around the Gradual Release of Responsibility, with a tune in, shared time and then collaborative work.

Tune In:

D used a warm up game to tune the students into the session. They played a game that was then analysed during the share time to introduce algebra. This was useful as the students came to the instruction part of the lesson, tuned and ready to learn.

Shared Time:

This time was used to give instruction. The lesson included an introduction to algebra yet focused on problem solving strategies. This meant that students had the skills to apply their knowledge to the problem. During this time D continually checked for understanding and clarified key points. When the students went to collaborate on their work, they understood what was expected and how to solve the problems.

Collaboration:

During this time D conferenced with the individual and groups of students. He used questioning as well as feedback to help guide students to think about and adjust their strategies. This was useful, as rather than telling the students what to do he prompted a collaborative discussion in which they came to their own conclusion. In this way they had to analyse and justify their thinking.

What I learnt:

The main point I took from this session was how to more effectively use feedback during conferences. I need to incorporate more explicit feedback in my discussions with students. I will continue to use discussions as a way to assess and scaffold student understanding, but I will also try and include more individual discussions, where students will be comfortable receiving explicit
feedback.

How it helped address the question for inquiry:
D differentiated using student choice and open ended tasks, resulting in the students being enthused and engaged by the task. Due to the success of the lesson I am going to incorporate these strategies more in my planning and implementation of lessons.

TALKING TO COLLEAGUES

Date: 12.8.2014
Teachers Involved: L
What was discussed:
The discussion was focused on how to structure a lesson when the learning intention and outcome is not uniform. For this situation, we discussed the benefit of structured focus groups, where the learning intention for each group could be different. I felt this suited the situation, as I was going to be teaching different content to different groups of students. Once I had decided to go ahead with this structure, L advised me on how to manage classroom expectations and behavior so that I was not interrupted when working with a group. She suggested strategies such as peer helpers and discussing and setting expectations with the class before the lesson.

How it helped address the question for inquiry:
The discussion helped me work out how to structure my lessons to ensure all learning needs and goals were catered to. This also helped guide my planning and sequencing of lessons. I used the strategies discussed to ensure that when I was working with a focus group, that I had uninterrupted time to scaffold their understanding.

Date: 15.8.2012
Teachers Involved: R
What was discussed:
We opened the conversation by discussing the learning goals of the sessions. Doing this helped clarify what I wanted the students to achieve. We discussed the benefits of expanding the depth of students' understanding before expanding the breadth. R then shared with me some ideas of how to do this, such as learning tasks with low floors and high ceilings, where students can enter the task at a level appropriate for their learning. He pointed me in the direction of some research and resources I could use, including Charles Lovitt and Jo Boaler. We then discussed how I would be able to assess for student understanding using the AusVELS proficiencies rather than progression points.

How it helped address the question for inquiry:
After this conversation I reconsidered the learning goals I had set. Using R's suggestion I looked at the AusVELS proficiencies rather than progression points. Using work samples I identified where the students were at with their proficiency and where I wanted to scaffold them to. For more detail see Appendix 3 on this development.

OTHER PROFESSIONAL LEARNING AND HOW IT HELPED ADDRESS THE QUESTION OF INQUIRY:

Author: Charles Lovitt & Doug Clarke (Appendix 2a)
This research looked at what makes a good mathematics lesson, particularly what elements need to be planned and used to create an outstanding experience. Lovitt and Clarke highlight the importance of student engagement and enjoyment. They discuss the need for student choice and ownership, challenging students, authentic tasks and use of technology among other things. This professional reading informed my action plan in many ways, one of which was the use of challenging tasks to engage students and give them ownership of their work.

Author: George Booker, Denise Bond, Len Sparrow, Paul Swan. (Appendix 2b)


This reading influenced the unit in many ways, particularly once I moved on to problem solving. Booker, Bond, Sparrow and Swan discuss the importance of problem solving to learning and understanding. Their discussion influenced my action plan as I moved towards using problem solving to teach and assess mathematical understanding. I used their assessment rubric to guide how I measured student achievement and understanding. They also address the complexities of introducing algebra to primary school students. The highlight that some students may struggle with the switch to symbols, and suggest using meaningful symbols and tasks to help the students transition.

Author: Jo Boaler (Appendix 2c)


In her article Boaler emphasizes the importance of moving away from rote learning. The research she discusses shows that students who simply learnt formulas and worked with text books were unable to apply their knowledge to problem solve in exams or real life situation. The research also found that those students who worked with authentic tasks that were open ended had a greater ability to retain and apply their content knowledge. This information guided the new learning outcomes for the unit. I am aiming to structure a unit in which the students learn in a meaningful way, so that they can adapt what they know and apply it to new situations.

**SECTION 3 - APPLICATION**

**APPLYING KNOWLEDGE TO PRACTICE**

**3A: THE ACTION PLAN**

**ACTION PLAN:** See Appendix 3

**CHANGES TO LEARNING OUTCOMES:**

The learning outcomes of the unit began by focusing on adding to the students’ content knowledge, or adding breadth. This was based on the progression points in AusVELS and only covered understanding and fluency and problem solving. After observation of the students and professional discussions, it became clear that whilst breadth of content is valuable, the deeper the understanding the more useful the knowledge is. Due to this, the learning outcomes were changed to focus on proficiencies, or depth, rather than breadth. The learning outcome after 15th of August was:

For students to apply their understandings to problem solve and justify outcomes. This specifically includes:

- Formulating and solving authentic problems, using number and measurement.
- Applying number laws to calculations.
These changes can be seen in the action plan in Appendix 3.

SPECIFIC RESOURCES:

The main resources used were:

- Mathletics
- Maths 300
- iPads

FEEDBACK TO STUDENTS:

During the focus group sessions and conferences there were many opportunities for me to give feedback to the students and for the students to peer assess. The one on one conferences were particularly useful in this way, as students were able to be supported by positive feedback, which was important for those requiring encouragement. I was also able to assist in the setting of learning goals, so that the success criteria were clear and students knew if they were meeting it. See Appendices 3 and 5 for more detail.

APPLICATION OF KNOWLEDGE:

The action plan and its evolution was heavily influenced by the new understandings I gained through observations, professional discussions, professional readings and self-reflections. The main ideas I have applied in my planning are:

- Focusing on depth of knowledge. This was achieved though explicitly teaching the students about the difference between depth and breadth. I encouraged and scaffolded students in explaining their thinking and using reasoning when applying their content knowledge. This extended into encouraging students to create ideas or problems using what they knew.
- Allowing for student voice and ownership. This was achieved by allowing students to problem solve using their preferred strategies and create their own problems. After reflection I decided I had not included enough student voice, so I endeavored to provide more opportunities as the term progressed.
- Informally checking understanding. This was done throughout the lessons and is now a more routine part of my daily practice.
- Use of open ended tasks to differentiate. Open ended tasks allow for a gradual release of responsibility from explicitly teaching a concept to students applying it independently. Once I began using this method more, my confidence in applying it developed and it became an important part of my practice.
- Using ICT in a meaningful way. The iPads were a valuable resource is making it easier for students to share ideas and demonstrate understanding. The applications used made it quicker for the students to create diagrams or presentations. They were also able to instantly share work via email and flick.
- Collaborating with colleagues. This was something that I was doing but not regularly. The observations and discussions informed my teaching and allowed for meaningful reflection. This is something I need to continue to build into my routine.
SECTION 4: IMPLEMENTATION

IMPLEMENTING THE ACTION PLAN

4A: PROVIDING FEEDBACK ON THE ACTION PLAN


SIGNATURE OF MENTOR TEACHER: L

SUMMARY OF DISCUSSION AND ACTIONS ARISING:

Observation 1:

In this session L observed a lesson which put into effect the decisions made during our discussion documented in Section 1. After the lesson we discussed what had been effective and what needed to be changed. Overall the groups had been successful and the learning goals had been achieved. However the following areas for improvement were discussed:

• Use of physical models. The concepts being taught were quite abstract and L noticed that some of the students seemed to ‘lose where they were up to’. She suggested that if physical models were used this might help the students visualize what was happening. We discussed ways of doing this including using numbered cards that the students could move about and in this way act out their thinking. I need to include more physical models in future lessons.

• Setting up expectations for the lesson. During the beginning of the lesson I did not remind students of the behavioral expectations when working in focus groups. Due to this the students would interrupt the focus group to ask questions, rather than using their peers or moving onto the next section. L suggested that in the future I remind students of how the group structures lessons runs before they break up into their groups. This is something I will plan for in future lessons.

Observation 2:

This session was an introduction to open ended problems and a focus on problem solving strategies. During the lesson the students were engaged and were managing the expectations for focus groups. In the discussion that followed the lesson, L and I reflected on how I had implemented the feedback from the previous observation, particularly setting behavioural expectations at the beginning of the lesson. We also discussed how much the students had enjoyed the hands on, outdoors nature of the session. The main items of feedback we discussed were:

• Student voice and choice. L highlighted that one of the things the students had really enjoyed in the lesson was the ability to choose how they would solve the problem. She asked me to reflect on the previous lessons through a lens of student voice. I realised that I had not been using this effectively if at all. With L’s support I looked at the action plan and revised it to include more voice and choice.

• Share time. We discussed that whilst I often plan for share time, it can easily be pushed aside to give students time to finish the task. L suggested that I ensure I include it because it gives an opportunity for further collaboration and reflection. We discussed ways to do this and decided that the method I had observed in J’s classroom would work well. Therefore in future lessons I will leave time for students to share their ideas, informally sitting at their tables. This is quick, but still allows students to share their thinking strategies.
Observation 3:

During this session the students were drafting their worded mathematics problems. This series of lessons was very challenging for the students, and many needed significant scaffolding. However, as L and I discussed, the students were engaged and really enjoyed being creative with mathematics. The main area for improvement was how I organised the differentiation. Because each student was working on a different problem and needed specific scaffolding, I was very pressed for time and not all students got the time required. We discussed that the benefit of the sessions is that the students have true ownership of their work when they are allowed complete autonomy. However, it becomes very difficult to cater to student needs with 27 entirely different questions. L had two suggestions for adaptations:

- Organise focus groups around type of question and keeping instruction general: This way, although the feedback might not be question specific, I could give general feedback to students about how to organise their problems. This would have been sufficient support for many of the students.

- Give students a choice of questions to develop. This would mean less autonomy for the students but would allow for more effective scaffolding. On reflection this seems to be the best option. I will include this idea into my future teaching as it caters to student voice and the requirements of explicit instruction.

SECTION 5: EVALUATION
EVALUATING THE EFFECTIVENESS OF PRACTICE.

5A: ASSESSING THE LEARNING OF STUDENTS

ASSESSMENT OF STUDENT LEARNING: See appendix 4 for pre and post testing and student work samples.

AREAS OF PRACTICE THAT NEED TO BE EVIDENCED:

Uses effective verbal and non-verbal communication strategies to support student learning.

C has used verbal communications effectively during whole class, small group and individual discussions. She also uses non-verbal communications to cater for EAL students and visual learners.

Involves parent/carer involvement in their child's learning.

C has maintained communication with parents during parent teacher interviews and phone calls to encourage parents to support their child's learning.

Ensures students' well-being and safety.

C has taught You Can Do It lessons to cater to student wellbeing. C has ensured that students are safe on excursions and camp.

Uses strategies to promote safe, responsible, ethical use of ICT.

C has taught Cyber safety lessons to ensure are safe and responsible when using websites and apps.
Demonstrates professional ethics and responsibilities.

Throughout this inquiry C has demonstrated this by caring for and respecting the students and their wellbeing as well as seeking advice when needed.

Complies with legislative, administrative and organisational requirements.

C has used AusVELS among other curriculum documentation to plan this unit.

Engages with teaching colleagues, professional networks or the broader community

C regularly completes Professional Learning in the PD form and readings form. She happily and regularly seeks advice from her colleagues about good practice.

5B: EVALUATING EFFECTIVENESS OF PROFESSIONAL PRACTICE.

OVERALL:

The implementation of the action plan was effective in that the learning outcomes were achieved. This can be seen in the students’ work and their change in attitude towards mathematics. For example they are more engaged and are happy to justify their answers. My scaffolding of how to apply mathematical concepts was effective, evidenced by the students ability to apply algebraic concepts to problem solve.

However I was not as effective is teaching the students why and how to explain their thinking. Many of the students still do their working out on a piece of paper that gets thrown away. The culture among them remains that the final answer is the most important thing and that it needs to be presented neatly and perfectly. This indicates I need to continue to scaffold their understanding of what they are doing and why. As part of this I need to include better organisation of work. One of the issues I found in assessing for student understanding is that the students tend to throw out their working or not keep worksheets or iPad presentations. I need to be aware of this and support students in organising their work and filing it.

STUDENT 1:

Student 1 has progressed in his ability to explain his thinking, problem solve and reason using mathematics. At the beginning of the unit he had a limited ability to explain his thinking, which can be seen in Appendix 4a. This was also evident from observations during discussions and conferences (Appendix 5). The post test, seen in Appendix 4d, demonstrates an understanding of problem solving and reasoning with the creation of a problem. The problem is not overly wordy, but includes high level mathematical thinking. Student 1’s answer to another student's problem also demonstrates an understanding of the problem solving process and an ability to reason using algebra and number. He is able to use diagrams to convey his understanding and thinking process, and used them to support his justification of his result.

Whilst Student 1 has developed his problem solving capacity and ability to reason using mathematics, at times he still struggles to apply this to new situations. Over the remainder of the year I will continue to work with him on broadly applying his skills.

STUDENT 2:

Student 2 has increased his capacity to problem solve, however he displays it inconsistently. His pre-test (Appendix 4e) shows that his content knowledge is a year ahead of standard, however his ‘Trip Around Australia’ project demonstrated that he was reluctant to display his working out and explain his thinking (Appendix 4f). This could indicate that he does not understand the importance of reasoning.
This was also evident during discussions and through observation. The problem he created showed some understanding of how to apply mathematical concepts to problems, however, discussions revealed that Student 2 had not thought deeply about the problem and had "just come up with something that looked hard". To continue to build Student 2's capacity to problem solve and reason using mathematics I will continue to build a classroom culture where justifying, reasoning and problem solving is the norm. My observations showed that Student 2 responds better to peer expectations and collaboration. If his peers are expecting to justify and reason in their work then this will have a positive influence.